

# Projective Pathways Towards Roitman's Model Hypothesis

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We report on ongoing work towards proving that the existence of Paul E. Cohen's pathways [1] implies the existence of a sequence of Roitman models of elementary substructures of  $H(\omega_1)$  [2]. Both notions have recently featured in work of Barriga-Acosta et al. [3] who exhibited a connection between pathways and sequences of Roitman models, which in turn feature in relation to the box product problem and the problem of  $P$ -points. We illustrate an approach to deducing Roitman's Model Hypothesis MH from Cohen's pathways using ideas from hyperarithmetical theory and higher computability theory [4]: by defining a stronger notion of pathways, which satisfy additional closure properties, and using techniques of effective descriptive set theory due to Moschovakis concerning the projective sets [5], we outline an approach to building an increasing sequence of elementary substructures of  $H(\omega_1)$  from sets of reals.

## References

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