Lipschitz-free spaces and De Leeuw representations

Richard J. Smith*

University College Dublin, Ireland
richard.smith@maths.ucd.ie

Let M be a complete metric space with base point 0. The Lipschitz-free (hereafter free) space $\mathcal{F}(M)$ over M is a Banach space whose dual is the space $\mathrm{Lip}_0(M)$ of real-valued Lipschitz functions on M that vanish at 0. Free spaces have many interesting properties and can be found at the interface between functional analysis, metric geometry and optimal transport theory. They are easy to define but hard to analyse, and have been studied intensively by functional analysts for over 20 years.

Recently, the set of extreme points of the closed unit ball of every free space was characterised, solving a problem that can be traced to the mid-1990s [1]. The solution relies on the De Leeuw transform, which enables elements of $\mathcal{F}(M)$ to be represented by Radon measures on the Stone-Čech compactificaton $\beta\widetilde{M}$ of $\widetilde{M} := \{(x, y) \in M^2 : x \neq y\}$. In this talk I will introduce the above and present some of the topological and measure-theoretic ideas (see [2, 3]) that were essential in resolving the extreme point problem.

References

[1] R. J. Aliaga, E. Pernecká and R. J. Smith, *A solution to the extreme point problem and other applications of Choquet theory to Lipschitz-free spaces*, arXiv preprint (2024), arXiv:2412.04312.

[2] R. J. Aliaga, E. Pernecká and R. J. Smith, *De Leeuw representations of functionals on Lipschitz spaces*, Nonlinear Anal. (260), 113851, 2025.

[3] R. J. Smith, *A Choquet theory of Lipschitz-free spaces*, arXiv preprint (2024), arXiv:2412.05177.

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